Russell's Logical Atomism 1914–1918: Epistemological Ontology and Logical Form

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Abstract

Logical analysis, according to Bertrand Russell, leads to and ends with logical atomism, an ontology of atomic facts that is epistemologically founded on sense-data, which Russell claimed are mind-independent physical objects. We first explain how Russell's 1914–1918 epistemological version of logical atomism is to be understood, and then, because constructing logical forms is a fundamental part of the process of logical analysis, we briefly look at what has happened to Russell's type theory in this ontology. We then turn to the problem of explaining whether or not the logical forms of Russell's new logic can explain both the forms of atomic facts and yet also the sentences of natural language, especially those about beliefs. The main problem is to explain the logical forms for belief and desire sentences and how those forms do not correspond to the logical forms of the facts of logical atomism.

According to Bertrand Russell all "philosophy is logic," by which he meant that "every philosophical problem, when it is subjected to the necessary analysis and purification, is found either to be not really philosophical, or else to be, in the sense in which we are using the word, logical" (OKEW, 42). By necessary analysis Russell meant logical analysis, i.e., formally breaking up a complex whole into its separate parts or constituents, and then explaining the way the parts are held together in terms of a logical form (OKEW, 52). Logical forms are themselves determined by a background logic, which is a formal system the grammar of which determines the logical forms of the system and the principles of which, usually consisting of axioms and inference rules, determine what is valid or provable in the system. In such a background logic, logical forms will have truth conditions and deductive consequences that are more perspicuous than they are in ordinary language. Russell's implicit background logic was his ramified theory of types.

1 On Logical Forms and Facts

A standard view of the complexes to which logical analyses are to be applied are the declarative sentences of ordinary discourse. That is certainly what Russell

assumed when he said that "some kind of knowledge of logical forms, though with most people it is not explicit, is involved in all understanding of discourse" (OKEW, 53). This is because "in order to understand a sentence it is necessary to have knowledge of both the constituents and of the particular instance of the form"(ibid.). Thus, by logical analysis on this view one clarifies or explicates the logical forms people implicitly understand in their use of language, thereby also clarifying the truth conditions as well as the logical implications of what they say or understand others to say to them.

But Russell also claimed that facts, or at least the atomic facts of logical atomism, have logical forms as well, and facts are part of the world, not of language (PLA, 183). Indeed, true atomic sentences, or what Russell called elementary propositions, are true because there are atomic facts in the world that they mirror in a one-to-one correspondence, and which therefore have the same logical form (PLA, 199). Such atomic facts provide the truth conditions for elementary propositions. But the atomic facts and elementary propositions that correspond to them in Russell's logical atomism are not the facts and propositions we talk about and use in our analyses of ordinary discourse. Russell's atomic facts and elementary propositions are the result of a different kind of analysis, and in particular one based on the immediate objects of experience.

In logic, or so Russell claimed, we "are concerned with the form of facts, the different logical sorts of facts there are in the world" (PLA, 216f). Indeed, according to Russell, "one might describe philosophical logic... as an inventory, a 'zoo' containing all the different forms that facts might have" (PLA, 216). One kind of fact in particular that Russell was concerned about are general facts. He argued, for example, that if you try to verify a universal sentence 'All A are B' by verifying the set of atomic propositions of the form 'x is an A and x is a B', you will have assumed that you have a complete list of the relevant atomic facts, and that that is but another universal sentence, and moreover one which you cannot verify in the same way (PLA, 236). That is true, but that is an epistemological argument regarding how we can verify a general statement, and, as such, it is independent of the ontological claim that there are general facts.

We should be clear here that knowing the atomic facts that correspond to elementary sentences suffices (in, e.g., model theory or Carnap's state descriptions) to explain the truth conditions of complex sentences of a formal language, including those containing quantifier phrases—or at least that is so for *extensional* formal languages. That means that the truth conditions of general statements are also determined by the atomic facts, and therefore Russell's suggestion that we might need general facts can be put aside and not taken seriously insofar as the truth conditions of general statements are in question. For more on this issue of facts in the world, see section 6 below.

Russell apparently thought that there are different kinds of facts in his ontological "zoo". In particular, he assumed that there are facts corresponding to belief statements, which of course are the kind of assertions we express in ordinary discourse. But there is a problem of explaining the structure of such facts, and problems also of whether or not their truth conditions can be accounted for

in Russell's logical atomism. We discuss this problem in section 5 below.

The correspondence between the logical forms of Russell's logic and the logical forms of facts brings in more than a theory of the logic of ordinary language, and in fact it brings in the ontology of logical atomism, and, as we will see in section 2, it is a fact ontology that is essentially based on a certain epistemological view of the foundations of knowledge. In section 3 we discuss how Russell tried to connect the facts of logical atomism with the facts of the physical world as ordinarily understood. In section 4 we explain how Russell's implicit background logic is not quite what he thought it was.

2 The Ontology of Logical Atomism

Russell called his framework logical atomism because it consisted of "many separate things" (PLA, 178), but there really is much more to Russell's logical atomism than that. Certainly, one could have an ontology other than Russell's logical atomism that consists of many separate things. Classical Greek atomism, for example, consisted of many separate things. But, in fact, Russell's atomism is not at all the same as Greek atomism. This will become clear as we explain the version of atomism that Russell had in mind in the years 1914–1918, which began with Russell's 1914 article "The Relation of Sense-Data to Physics" (SDP), chapters 2–4 of his 1914 book, *Our Knowledge of the External World* (OKEW), "The Ultimate Constituents of Matter" (1915), and then was developed more in his 1918 lectures, "The Philosophy of Logical Atomism" (PLA). Russell also described a modified version in his 1924 paper, "Logical Atomism" (LA), but we are mainly concerned here with the 1914–18 version of Russell's logical atomism.

The many separate things of Russell's logical atomism as an ontology are referred to by Russell as simples and complexes. The complexes are called atomic facts (and possibly some general facts and belief facts), and the simples are called particulars, qualities and relations. Atomic facts are composed of particulars having qualities or standing as the terms of a relation. All of the remaining things in the world are represented in terms of these simples through a process of logical analysis and construction, the result of which all other things, including the objects of mathematics, are or can be considered as "logical fictions." These logical fictions "embrace practically all of the familiar objects of daily life: tables, chairs, Piccadilly, Socrates, and so on"(PLA, 253). Just this sort of ontological project explains why Russell claimed that the "business of philosophy is essentially that of logical analysis, followed by logical synthesis" (LA, 341).

Particulars, according to Russell, are the "terms of relations in atomic facts."[1] But these particulars are not, as we have said, the "atoms" of classical Greek atomism, i.e., the atoms of Leucippus and Democritus. Atoms in classical Greek atomism are the smallest bits of matter that in a completely random way clump together in a void to make arbitrary larger clumps of matter, some of which in

[1] PLA, pg., 199. Apparently, qualities in atomic facts with a single term are also called relations by Russell.

time ultimately make up suns, planets, people, and all of the familiar objects of daily life. Russell's particulars, on the other hand, were the "objects of sense," e.g., patches of color, sounds, and smells. He called these objects sense-data and sometimes *sensibilia*. In general, sensibilia are "those objects which have the same metaphysical and physical status as sense-data, without necessarily being data to any mind" (SDP, 148). Sense data are "not mental," but in fact are "part of the actual subject-matter of physics" (DSP, 149). Russell's atoms, in other words, are the atoms of a kind of phenomenalism, except that they are also said to be "physical," and Russell's phenomenalism neither assumes nor denies the existence of other physical objects.[2] Sense-data are the particulars that have qualities and stand in the various relations of the atomic facts that make up Russell's ontology. But even more radical, at least in his 1914 work, Russell allowed for the possibility of unsensed sensibilia. Unsensed sense data, just as sensed sense data, were said to be "physical," not mind-dependent, entities, which is why they can exist even if unsensed (SDP, 151f). Each particular sense datum, according to Russell, "has its being independently of any other and does not depend on anything else for the logical possibility of its existence" (PLA, 203), including minds. An epistemology based on sense data might perhaps be a reasonable starting point for knowledge about the world, at least for some philosophers, but an ontology based entirely on sense data is another matter altogether—unless of course one is a subjective idealist or a solipsist. But Russell was quite explicit that his version of phenomenalism was definitely not subjective idealism, and the independence of sense data as physical objects precludes solipsism as well.

As physical objects, unsensed sense-data are actual physical objects and not merely possible physical objects. Unsensed sense-data and *possible* sense-data that are not *actual* sense data are different ontological kinds of sensibilia. A merely possible sense-datum is a possible physical object that *would* exist, i.e., be an actual physical sense-datum, if a mind *were* to be at a certain perspective where that mind *could* be acquainted with that sense-datum. Such a possibility presupposes the reality of physical space and, apparently, the reality of a mind that could be acquainted with sense-data, and also a modality that would be the basis of subjunctive conditionals regarding being acquainted with merely possible sense-data. But, for reasons connected with Occam's razor, Russell claimed that he could construct a counterpart, i.e., a logical fiction, of real physical space and similarly a logical fiction counterpart to a mind. Perhaps that is why Russell assumed that unsensed sense-data were *actual* physical objects and not merely possible objects. An analogy here is with Bishop Berkeley's famous problem of whether or not a falling tree makes a sound in a forest when no one is around. For Berkeley the answer is yes because God is always around to hear the sound of the falling tree. For Russell, on the other hand, there is no god, but the sound impression still exists as an unsensed sense datum, a physical object that exists independently of actually being sensed.

[2] A discussion of sense data as physical objects can be found in Russell's 1915 paper "The Ultimate Constiuents of Matter", reprinted in Mysticism and Logic.

On the other hand, it is not clear what it means for sense-data to be physical objects in logical atomism, given that physical objects are "logical fictions" to begin with (because the notion of a physical object is a notion constructed in logic[3]), suggesting that sense-data must then be logical fictions as well. It is dubious, however, or at least not clear that Russell ever had any of these considerations or problems in mind in his account of unsensed sense-data, and it is an ontological category that he simply doesn't mention again after 1914–15.

In any case, sense-data "are all that we directly and primitively know of the external world" (SDP, 148), according to Russell. They are for that reason the foundation of all knowledge. This is an epistemological doctrine, and as such it is the basis of the phenomenalistic ontology of Russell's logical atomism. In this phenomenalism, Russell does not deny the existence of the "familiar objects of daily life." Rather, he says that we need not assume their existence if we can logically construct their counterparts as "logical fictions."

Sense-data are what "names in the logical sense" denote, where names in the logical sense are names like 'this' or 'that' (OKEW, 83). According to Russell, "it is only when you use 'this' quite strictly to stand for an actual object of sense that it is really a proper name" (PLA, 201). It is because particulars are objects of sense that Russell speaks of the "qualities" of particulars and not of their "properties". "Knowing a particular merely means acquaintance with that particular" and its qualia (204); and, moreover, we can be acquainted only with objects of sense.

In his 1910–13 framework (of PM and PoP), Russell did say that we can be acquainted with facts as well as particulars, facts such as my-seeing-the-sun, or an-object-a-having-the-quality-q, and a-in-the-relation-R-to-b. At that time, Russell used perfect nominalizations of sentences to denote such facts, which then became "terms" that could be arguments of predicates. In PM, for example, Russell said that among the particulars with which we can be acquainted there are also complexes such as this-above-that, the-yellowness-of-this, etc.

But in 1914 and thereafter, Russell no longer allowed nominalized sentences to occur as terms of relations. Thus, according to this later Russell, "you can never put the sort of thing that makes a proposition to be true or false in the position of a logical subject " (PLA, 188). In other words, in Russell's logical atomism we can be acquainted only with particulars, namely, sense-data. Moreover, if the existence of the atoms of modern physics are to be verified, according to Russell, "it must be solely through their relation to sense-data" (RSDP, 145.) This issue about nominalized propositional forms is really about logical form, a topic we take up in section 5 of this essay.

It is noteworthy that Russell also called particulars events, and in 1912 he called events facts (PoP, 49). But in 1914 and thereafter, events are not facts but sense-data. This is because, according to Russell, sense-data are the objects of sensations, and sensations are mental events. But in 1914 and for some time thereafter Russell assumed that a sense-datum is identical with a sensation

[3] See, e.g., UCM page 128.

(OKEW, 83).[4] From that assumption it follows that a sense-datum is an event. This claim, incidentally, is contrary to what Russell wrote in 1912 (PoP, 12). Anyway, given that sense-data are events, we can see why Russell would later write that each of the simple entities of the world "may be called an event" (LA, 341). In other words in Russell's atomism bits of matter "are not among the bricks out of which the world is built. The bricks are events... "(LA, 329).

3 The Bridge Between the World of Sense and the Physical World

The bridge between the physical world and the world of sense is to be built, according to Russell, by means of a logical construction within the logic of logical atomism. The task "requires a very great amount of logical technique" and is a very difficult one to carry out. The idea is that for each kind of thing in the physical world we are to "construct a logical fiction having the same formal properties, or rather having formally analogous formal properties to those of the supposed metaphysical entity" (PLA, 272). The construction can then be substituted for the "supposed metaphysical entity and fulfil all the scientific purposes that anybody can desire" (ibid.). In speaking of a certain desk, for example, as being the same through a period of time, all we "can know is that there are a certain series of appearances linked together, and the series of those appearances" is essentially what Russell defines as the desk (PLA, 273). "In that way," according to Russell, "all the ordinary objects of daily life are extruded from the world of what there is, and in their place as what there is you find a number of passing particulars that one is immediately conscious of in sense" (ibid.).

The construction is only briefly sketched by Russell, and he emphasizes that he is "only constructing a *possible* theory" (OKEW, 95). The idea is that from moment to moment "each mind looks out upon the world ... from a point of view peculiar to itself" (ibid., 94). "The system consisting of all points of view of the universe, perceived and unperceived," is called the system of perspectives (OKEW, 95). The "unperceived" perspectives involve unsensed sense-data. A momentary common-sense "thing" appears from an uncountable number of perspectives, not all of them actually sensed. Some of the perspectives, e.g., of a penny, are in conflict with other perspectives; e.g., some being round, some elliptical, some are flat, etc., all from different positions. The momentary constructed "thing" is then identified by Russell with a class of perspectives that are somehow selected at that moment from the totality of perspectives by means of the multiplicative axiom (which in type theory is equivalent to the axiom of choice). Here, incidentally, we note that the multiplicative axiom is needed only because unsensed sense data at the infinitely many different perspectives are actual physical objects, and, supposedly, an infinite subclass of perspectives is

[4] Ultimately, Russell gave up the idea that sensations are mental (as opposed to neurological) events. See MPD, 136.

to be selected. But still, Russell does not say who, or what, applies the multiplicative axiom. It cannot be the mind of a person whose perspectives are at stake, because there is not always a person at those perspectives. And it cannot be some metaphysical aspect of the universe or of Russell's ontology, or at least not without some further explanation. None of this is explained. Nor does Russell explain much of his construction at all, and in fact the whole process of construction in this way seems quite dubious.

A more plausible logical construction has been given to some extent by Rudolf Carnap in his 1928 *Der Logische Aufbau der Welt (The Logical Construction of the World).* One important difference is that for Carnap there are no unsensed sense-data, and the "atoms" of his construction are elementary experiences (*Erlebnisse*) and not logically independent sense data, which indicates that Carnap's version of phenomenalism is different from Russell's.[5] In particular, Carnap does not assume that elementary experiences are mind-independent physical objects, and it is important to note that he does not use the axiom of choice at all. He also calls his framework methodological solipsism. In a way, Carnap's solipsism is like Wittgenstein's in the *Tractatus,* according to whom "solipsism, when its implications are followed out strictly, coincides with pure realism. The self of solipsism shrinks to a point without extension and there remains the reality coordinated with it" (Tract. 5.64). Finally, another difference is that Carnap's logic is his version of simple type theory where, unlike Russell's implicit logic in this period, predicate expressions can be terms or arguments of higher-order predicates. This difference is explained more in section 4 below.

All of the concepts of science, according to Carnap, can be analyzed and reduced to certain basic concepts that apply to the content of what is given in experience. One of the important patterns for such an analysis is known today as *definition by abstraction*, whereby, relative to a given equivalence relation (i.e. a relation that is reflexive, symmetric and transitive), certain concepts are identified with (or represented by) the equivalence classes that are generated by that equivalence relation. This pattern was used by Frege and Russell in the analysis of the natural numbers (as based on the equivalence relation of equinumerosity, or one-one correspondence), and was then used again by Russell and Frege in the analysis of the negative and positive integers, the rational numbers, and even the real numbers. In addition, Carnap, who took definition by abstraction as indicative of the "proper analysis" of a concept, generalized the pattern into a form that he called "quasi analysis" (but which really amounted to a form of synthesis), that could be based on relations of partial similarity instead of full similarity, i.e., on relations that amount to something less than an equivalence relation. An example of a partial similarity is that between the *c-e-g* musical chord and other chords containing c but not either e or g, or e but not either a or g, etc.

The concepts definable in terms of a quasi-analysis specify in what respect

[5] In the 1961 preface to the English translation of his *Aufbau* Carnap does say that he would prefer using Ernst Mach's terminology 'concrete sense data' instead of 'elementary experiences' for the basic elements of the system. But there are no unsensed sense data or unexperienced elementary experiences in Carnap's framework.

things (especially items of experience) that stand to one another in a relation of partial similarity agree, i.e., the respect in which they are in part, but not fully, similar (see Carnap 1928, sections 71-74). In this way Carnap was able to define the various sense modalities (as classes of qualities that intuitively belong to the same sense modality, where concepts for sense qualities are definable in terms of a partial similarity between elementary experiences), including in particular the visual sense and color concepts (as determined by the three-dimensional ordering relation of the color solid).

Using the various sense modalities, Carnap went on to construct the four-dimensional space-time world of perceptual objects, which, with all its various sense qualities, "has only provisional validity", and which, for that reason, "must give way to the strictly unambiguous but completely quality-free world of physics" (Carnap 1928, 207). We cannot go into the details of Carnap's "constructional definitions" here, but we might note that all that Carnap meant by a logical analysis by means of such definitions was translatability into his constructional language—i.e., into an applied form of the simple theory of logical types as based on a primitive descriptive constant for a certain relation of partial similarity between elementary experiences. Such a translation need not, and in general did not, preserve synonymy, nor did it in any sense amount to an ontological reduction of ordinary physical objects to the sensory objects of experience. What it did preserve, according to Carnap, was a material equivalence (i.e. an equivalence of truth-value) between the sentences of ordinary language, or of a scientific theory, and the sentences of the constructional language.

Carnap was quite clear, incidentally, that his work was inspired by Russell's description of his view of a logical construction of the world in terms of a foundation built up in terms of experience. The question as to what extent Russell could accept Carnap's construction is problematic, as we will see, because for reasons having to do with the issue of logical form, Russell in this period could not allow the higher stages of Carnap's construction that go beyond second-order logic.

4 Logic and Logical Form in Russell's Logical Atomism

We should not think that the logical framework Russell assumed in his logical atomism is the ramified theory of types that he, along with Alfred North Whitehead, developed in their *Principia Mathematica*. As I have explained in some detail in a previous paper (the details of which we will not go into here), the logistic system Russell is committed to in his logical atomism is a fragment of second-order predicate logic, where the (first-order) predicate constants stand for the simple qualities and relations of different "degrees."[6] It is important to note here that the different degrees means only the different number of terms

[6] See Cocchiarella 1987, chapter 5, or Savage 1989, 41–62. We note that Russell seemed unaware that he could no longer use his original type theory for his atomist logic.

that first-order predicates may take as arguments (PLA, p.206f).[7] Third- and higher-order type theory is excluded in Russell's logical atomism because that would require that predicates (and relations), or (capped) propositional functions, can be terms or arguments of higher-order predicates, and that possibility is excluded by Russell's atomist ontology.[8]

Thus, according to Russell, "a predicate can never occur except as a predicate" (PLA, p.205). Similarly a "relation [qua expression] can never occur except as a relation, never as a subject" (PLA, P. 206). Russel's explanation is that the "different sorts of words, in fact, have different sorts of uses and must be kept always to the right use and not to the wrong use, and it is fallacies arising from putting symbols to wrong uses that lead to the contradictions concerned with types" (PLA, 206).

Russell is correct that the functional role and use of a predicate or relation (qua expression) grammatically cannot occur as a subject of a predicate or term of a relation. Nevertheless, both predicates and relations (qua expressions) can be and are transformed in natural language by a process called nominalization, and they then can occur as terms or arguments of predicates and relations. Thus the predicate 'wise' as in 'Socrates is wise' can be nominalized into 'wisdom', and then occur as a subject, as in 'Socrates exemplifies wisdom', where 'exemplifies' is a two-place predicate with 'Socrates' and 'wisdom' as (subject) terms. Similarly 'triangular' and 'rectangular' can be transformed into 'triangularity' and 'rectangularity', and verbs such as 'loves' and 'runs' can be nominalized as verbal nouns such as 'loving' and 'running', or as infinitive phrases as in 'to love' and 'to run'. The relation of identity as in 'is identical with' can be similarly nominalized as 'identity', and used as a term as in 'identity is a two-term relation'. For complex predicate expressions, which Russell called propositional functions (qua expressions), we would need a variable-binding operator such as Russell's cap notation, as in $\varphi(\hat{x})$, or Church's λ-operator before we could properly nominalize such a predicate expression, and adding such is not at all problematic. Accordingly, once we realize that nominalization is a standard procedure of natural language—and one that Russell himself engaged in prior to his adoption of logical atomism—then the issue of grammatical correctness is not itself a reason to exclude this nominalized treatment of predicates as Russell claims.

But there are at least two important issues that must be dealt with before we can allow adding nominalized predicates and relations to our logic. The first has to with whether or not the use of nominalized predicates would lead to a contradiction, as Russell claimed. One immediate response is Russell's own earlier logic of ramified types, which included the use of nominalized predicates or propositional functions as arguments or terms of third- and higher-order logic. Russell

[7] Russell's use of the word 'order' here means adicity, as an nth order predicate is really an n-adic predicate. Normally, by 'higher-order' predicate logic we mean a higher type level. The two notions of order should not be confused with one another.

[8] Russell's fragment of second-order predicate logic could be extended to ramified second-order predicate logic so long as predicate quantifiers for the extended logic are given a substitutional interpretation. See Cocchiarella 1980.

seemed to be simply ignoring or overlooking this use of nominalized predicates or propositional functions in his own type theory.[9] The important restriction of type theory (simple or ramified) is that predicates are distinguished according to type so that nominalized predicates or propositional functions of one type can occur as arguments or terms only of predicates or propositional functions of a higher type. Such a restriction succeeds in avoiding the paradoxes Russell had in mind in this context.[10] In addition, as I have shown elsewhere, one need not turn to third- and higher-order logic but can instead remain in second-order logic with nominalized predicates allowed so long as either certain metalinguistic restrictions are imposed on complex predicates (as λ-abstracts) as in my logic of λHST*, or otherwise we modify our first-order logic to a logic free of existential presuppositions regarding singular terms (λ-abstracts, in particular, occurring as terms), as in my logic HST$^*_\lambda$.[11] Both of these logics are equivalent to the simple theory of types, which is known to be consistent. Russell doesn't discuss the matter at all but simply assumes in PLA that nominalized predicates or (capped) propositional functions cannot be subjects of other predicates or propositional functions, higher-order or otherwise.

The second important issue about such a transformation is just what does a nominalized predicate denote as a singular term. In his earlier work, especially his *Principles of Mathematics* (PoM), Russell took the occurrence of nominalized predicates or relations as terms to denote the same properties and relations that they stood for in their functional roles as predicates and relations. Thus, for example, Russell assumed that 'Socrates is wise' and 'Socrates has wisdom' are formally equivalent, because the predicate 'wise' stands for the same property as the abstract singular term 'wisdom'. But, without appropriate restrictions, the result of allowing nominalized predicates led to a contradiction, specifically to what is now known as Russell's paradox (of predication). This interpretation can also be given for the first of the second-order logics mentioned above that I have developed, namely the logic λHST^*, which provides an appropriate restriction, and which then results in a consistent reconstruction of Russell's views in PoM, including his controversial treatment of plurals in terms of classes as many.[12]

Frege, as I have explained elsewhere, recognized the transformation of predicates and relations into nominalized forms in his second-order logic, and he took their denotata to be their extensions (*Wertverläufe*).[13] A definition of membership in a class could then be defined in terms of predication, and the

[9] Perhaps it was because he was never explicit about the transformation of the predicative role of predicates and propositional functions into terms or arguments of higher-order predicates and propositional functions. Or perhaps, as we note below, it was because in PoM and then in PM he took a nominalized predicate to denote the same universal that the predicate in its predicative role stood for.

[10] Russell's ramification of type theory was designed for certain semantic paradoxes as well, which do not concern us here.

[11] See Cocchiarella 1985, 1986 chapter 4, or 1987, chapter 4.

[12] See Cocchiarella 2002 for a consistent formulation and development of the logic of classes as many, which I have also used in my treatment of the logic of mass nouns.

[13] See Cocchiarella 1985 and 1987, chapter 4.

result was that Frege's logic contained a logic of classes (in the logical sense) as part of his predicate logic.[14] The result, without appropriate restrictions, was of course that Russell's paradox could then be derived, which meant that Frege's logic was inconsistent. But the second alternative of second-order logic with nominalized predicates that I developed and mentioned above, namely, HST^*_λ, provides a consistent reconstruction of Frege's logic (but now based on a free first-order logic).[15]

But the properties and (intensional) relations of Russell's 1903 *Principles* cannot be accommodated in Russell's logical atomism—nor can the extensions (*Wertverläufe*) of Frege's logic, or the sets of set theory. Nevertheless, we maintain that there cannot really be a "logically perfect language" for logical atomism given Russell refusal to allow for nominalization and all its uses in that language. This is because Russell's phenomenalism, as based on a physicalist interpretation of sense-data, is not a suitable framework for abstract entities of any sort. In other words, neither classes nor properties and relations can be allowed in Russell's ontology of logical atomism. Russell did not seem to be aware of how his atomist ontology truncated his logic. In particular, Russell can no longer construct the logical fictions of mathematics in his truncated logic. Nor can he construct the bridge between his ontology and the physical world.

5 Logical Form and Intensionality

Regarding the nature of belief, Russell asked: "What is the form of the fact which occurs when a person has a belief" (PLA, 217). Here we might note we are back to the issue of the logical analysis of ordinary discourse rather than the logical construction of the world in terms of logical atomism. We are also back to the issue of complex facts, including facts that "occur" when people have beliefs.

In turning first to the facts of belief we should first recognize the distinction between dispositional belief and occurrent belief. There is no single fact or event that is associated with dispositional belief; rather, there are a number of events indicative of what a person believes, including that person's verbal statements about his/her belief. This behaviorist notion of belief was familiar to Russell, and in fact he later came to adopt a modified (or supplemented) form of behaviorism as his own view. But in PLA, Russell rejected behaviorism and insisted that one must be dealing here with a new form of fact as a new

[14] By classes in the logical sense, I mean a theory of classes that have their being in the concepts whose extensions they are. The sets of ZF set theory have their being in their members, not in concepts. Indeed, there may very well be many more sets than there are concepts. Set theory is a bottom-up theory, whereas a logic of classes as the extensions of concepts is a top-down theory.

[15] I have adopted and developed a different interpretation of this logic based on a conceptualist theory of predication. Nominalized predicates denote the intensional content of the concepts (qua cognitive capacities) whose content they are. As based on the institutionalization of nominalization in the evolution of language, this is a mitigated version of intensional realism. See, e.g., Cocchiarella 2007.

kind of logical form.

Occurrent belief, on the other hand, is more a matter of either asserting verbally or thinking internally. In either case, it is a person's mental act, which might or might not be expressed verbally. Consider Russell's example of someone asserting 'I believe that Socrates is mortal'. How does this differ from when that person is simply asserting 'Socrates is mortal'? The latter is a statement about Socrates and the fact in question is that Socrates is mortal. Asserting the sentence 'I believe that Socrates is mortal' seems to be about the person and his or her beliefs, but, normally, people don't speak about their beliefs in this way. Rather we say things like 'I believe in democracy' or 'I believe in justice for the innocently accused', etc. Belief as normally expressed in this way does not involve two verbs, namely 'believe' and the verb of some proposition expressing our belief. Something like this was Russell's later view in LA where he said of propositional attitudes like first-person belief that "these attitudes do not form part of the proposition, i.e., of the content of what is believed when we believe, or desired when we desire" (LA, 309).

Of course, the problem is that we do assert third-person sentences such as 'Othello believes that Desdemona loves Cassio', to use Russell's example, but which for convenience we will abbreviate as 'A believes that B loves C'. In this case we do have two verbs, namely 'believes' and 'loves'. Russell claimed that "it is not only the proposition that has two verbs, but also the fact, which is expressed by the proposition, has two constituents corresponding to two verbs" (PLA, p.217). The problem is how is such a fact is to be represented in terms of the logical forms of Russell's logical atomism.

One rather standard analysis of this sentence is to represent belief as a relation between A and the proposition that B loves C:

$$Believes(A, that\text{-}P),$$

where *that-P* is the sentence 'B loves C' nominalized (so that it can occur as a term of the relation). But in Russell's logical atomism, along with all other abstract entities, "propositions are nothing" (PLA, p.223), and therefore nominalized sentences are nothing as well so they cannot be allowed, and hence this analysis cannot be given in Russell's framework. Similarly, Russell's analysis in terms of his earlier multiple relations theory of belief is excluded as well, because it involves nominalizing the predicate 'loves' as a term of belief in a four-term relation:

$$Believes(A, B, Loves(\hat{x}, \hat{y}), C),$$

where, using Russell's cap-notation, $Loves(\hat{x}, \hat{y})$, read as 'to love' or 'loving', is a nominalized form of the verb 'loves'. As already noted, (nominalized) predicates are not to occur as terms in the formal language of Russell's logical atomism. So this analysis is also excluded.

Another alternative is analyzing belief as an intensional operator that is indexed by the term A, which we will formally represent by $Believes\text{-}that_A$, and which is read as *A Believes-that*.[16] Note that on this analysis the relative

[16] This approach was first given by Jaako Hintikka 1962.

pronoun 'that' occurs as part of the operator and not as part a nominalized sentence. The result of applying this operator to a propositional form results in another propositional form (the way that the modal operator $\Box$ for necessity functions):

$$\textit{Believes-that}_A(B \textit{ loves } C).$$

Here the propositional form B *loves* C is not nominalized (thereby indicating that the verb *loves* occurs as a verb). Of course, adding such an operator means modifying Russell's logic and formal language for his version of logical atomism from an extensional logic to an intensional logic (or even a hyperintensional logic). Such an extension would allow for logical forms of sentences containing other intensional verbs such as 'desire'. That would answer Russell's query about how a logical analysis of such sentences could possibly be given.

The problem with this and other similar proposals is the question about what sort of new fact corresponds to the logical form that results from such a logical analysis. Russell's query in particular is: "What is the form of the fact which occurs when a person has a belief" (PLA, 217), and similarly when a person desires that something be the case. It does not seem to be like an atomic fact that might correspond to an elementary sentence. Yet, we want to suggest that it just might be what corresponds to a subject-predicate sentence nevertheless, but not one with a simple subject or a simple predicate, and not one that represents an atomic fact of Russell's logical atomism.

The point is that a modal or intensional operator that applies to a propositional form and results in another propositional form when applied in a particular case is equivalent to applying (predicating) an intensional (higher-order) property to a proposition, and hence in a sense it is analogous to asserting a subject-predicate sentence. What it then represents is not an atomic fact, of course, but if, as Russell insists, it represents a fact at all, as his query suggests, then what it represents is an abstract fact, which is not really part of the ontology of logical atomism. In other words, for A to assert that B loves C, is equivalent to A asserting that the proposition that B loves C has a certain intensional (higher-order) property represented by the operator $\textit{Believe-that}_A$, and therefore the fact, if any, that corresponds to such an assertion (as its truth condition) is an abstract fact. Such a result would suggest that we cannot escape having propositions and maybe even abstract facts as part of Russell's ontology of logical atomism if we are to make sense in that ontology of intensional verbs.

6 Some Assumptions and Consequences of Russell's Ontology of Facts

We initially started this essay by noting that according to Russell "some kind of knowledge of logical forms ... is involved in all understanding of discourse" (OKEW, 53). We take this to mean that logical forms underlie our thoughts, i.e., our judgments, as well as the sentences we use or statements we make in expressing those thoughts. This, as we noted, is the standard view of the

application of logical analysis. It is a view that goes back to the medieval terminist philosophers, and probably to Aristotle and Aquinas as well. In other words, for philosophers, and perhaps some linguists as well, the study and use of logical analysis and logical form is the fundamental and perhaps most important part of the study of language and thought. We take this to be a fundamental assumption of Russell's logical atomism.

For Russell, however, logical analysis and logical forms apply not only to the study of language and thought, but to the facts that make up the world, and therefore for all the other things that are based on those facts. In other words, according to Russell, philosophical logic is concerned not just with the logic of language and thought, but also with describing the world in terms of an inventory or zoo "containing all the different forms that facts might have" (PLA, 216).

A second basic assumption of Russell's logical atomism, accordingly, is that each fact in the world has a logical form. The logical forms of atomic facts, in particular, consist either of an object and a quality or of n-many objects and an n-place relation, for each natural number n. In Wittgenstein's version of logical atomism the world consists only of atomic facts, which, as already noted, suffices to account for the truth conditions of all of the sentences of an extensional formal language. But in Russell's version there are other kinds of facts as well, with other kinds of logical forms.

Now turning first to atomic facts, the main question is what accounts for their unity, i.e., what unifies the constituents or components of an atomic fact? The answer for Russell cannot be the so-called "tie" or relation of exemplification that many philosophers claim as the basis of predication, because that would require qualities and relations to be objects on a par with the objects that have those qualities or stand in those relations, as in the expression 'x exemplifies y', or '$x_1, ..., x_n$ exemplify$_n$ y' (where 'exemplify$_n$' is the the n-place "tie" that is the counterpart of the two-place "tie" 'exemplify'). Wittgenstein's alternative was a quasi-physical counterpart to Frege's notion of unsaturatedness (*Ungesätigeit*) and that is an essential aspect of every relation that is the nexus of an atomic fact.[17] In both cases, however, what is defended is a metaphysical notion that must be taken as primitive and left unexplained, except perhaps by hints (as in Frege's case) or in terms of a so-called language known as Jumblese.[18] Russell himself, at least in the 1914–18 period never says how the unity of an atomic fact is to be explained. He simply ignores the issue, except to say that relations are what hold a relational fact together, but he does not explain how relations do this, nor what holds a monadic fact together where there is no relation involved (and calling qualities relations will not do).

A third basic assumption of logical atomism is that facts in general are truth-makers; in other words, facts are the ontological ground of the truth-conditions of our judgments and statements about the world. This was clear in our observation that true atomic sentences are true in logical atomism because

[17] What is unsaturated nexus of atomic monadic facts was an issue in much of the later literature on Wittgenstein's logical atomism.

[18] See Sellars [1962] for a description of Jumblese.

they "mirror" atomic facts, and therefore they have the same logical form as the facts they mirror. This feature of atomic facts is related to the fourth basic assumption of logical atomism, namely, that the logical form of a fact, complex or atomic, corresponds to the logical form of the sentence that it makes true, or false as the case might be. If this were not so, then one fact would no more correspond to a true sentence than another fact. In other words, if a fact did not have a logical form that corresponds to a sentence that has that form, then there could be no correspondence between it and any sentence that it makes true. It is because facts have logical forms that they can be the truth makers of the sentences to which they correspond.

What this means is that logical forms are a fundamental feature of the world, even though they are not "discovered" through empirical investigations of natural science but rather only through logical analysis of language and thought. Moreover, because logical structures relate logical forms with each other in relations of logical consequence, then the structure of logic itself must be part of the structure of the world. This means that the question of which logic is the "correct" logic, i.e., the logic that correctly describes the logical forms that facts have in the world would seem to be a matter of which logic best accommodates the analysis of language and thought. There is no other way to form an "inventory" of the facts that make up the world.

One significant consequence of this result is that Russell is now committed to either adopt a formal logic more in line with language and thought, which would include a sublogic of nominalized predicates, a tense, modal and deontic logic, a logic of plurals and mass nouns, and perhaps other features such as a logic of questions, commands, performative verbs, etc. of natural language that either have already been, or might yet be, developed. If Russell could not give up his more restrictive extensional logic, then he must give up the idea of there being complex facts and allow only for atomic facts as the truth-makers of atomic sentences, with the truth values of all other formulas of his formal logic then being thereby determined (as described in model theory or Carnap's state descriptions). In particular, he must then give up the idea that there is a complex fact corresponding to a judgment of the form 'A believes that B loves C'.

7 Concluding Remarks

What did Russell do? Russell did not give up his extensional logic, but he did give up assuming that there is some one fact corresponding to a judgment of the form 'A believes that B loves C'. In 1919, only one year after he gave his definitive account of logical atomism, Russell modified his view both of propositions and of behaviorism. In particular, he adopted a version of behaviorism that he supplemented with an imagist theory of meaning in which propositions are complex images.[19] Image propositions are in no sense abstract entities, but there is more to *expressing* a proposition than having a complex image. Other

[19] See Russell 1919.

animals can have complex images, but in general they (and we) are not *expressing* propositions whenever such images occur. And even granting the notion of an image proposition, Russell's does not explain whether or not an image proposition can function as a term, and hence solve the problem of representing the logical form of judgments of what others believe. They are also problematic because images are subjective, not objective, entities, and as such they cannot really suffice for interpersonal meaning and communication. In particular, one person cannot compare his/her image proposition with another person's image proposition so that we could know that they are *expressing* the "same" proposition (when they occur in different people and therefore are really different images). Also, it is dubious that we can form images of things in the microphysical world smaller than the wavelength of light—things that we understand almost solely by mathematical formalism. It is also dubious that we can form and communicate images that denote or stand for abstract entities, regardless whether or not there are such entities. Imagist theories of meaning were popular in medieval philosophy, but they have long since been rejected by both linguists and philosophers.

Russell does not say how introducing image propositions answers the question of what logical form a judgment of the form 'A believes that B loves C' has, unless, of course, he assumes that an image proposition of the sentence 'B loves C' can be a term in a relation of belief. But in fact he must reject this answer because image propositions are themselves facts, and facts cannot be terms of relations in Russell's logic. So we still do not know what Russell takes to be the logical form of this judgement. We know only that his modified account of behaviorism is supposed to provide the truth conditions for the judgment.

Another important change occurred in the ontology of Russell's 1924 version of logical atomism. In the 1924 version logical constructions are no longer based on sense-data as the ultimate simples. The "bricks out of which the world is built" are still events, but they are not sense-data events. In Russell's 1924 "outline of a possible structure of the world," the world consists primarily of events, and all other objects, including minds, are to be constructed in terms of sets of so-called "compresent" events.

Russell moved from his 1914 view of the ultimate simples of the world as physical sensibilia, which included unsensed as well as sensed sense-data, to his 1918 view of mentioning only sensed sense-data, and then finally in 1924 not mentioning sense-data at all. Instead, simples are finally objects not experienced as simples "but known only inferentially as the limit of analysis"(LA, 337). In addition, according to Russell, "science has a much greater likelihood of being true in the main than any philosophy hitherto advanced" (LA, 339).

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